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Ontario Mathematics Gazette

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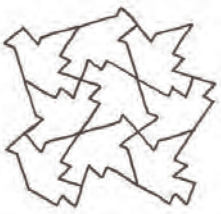
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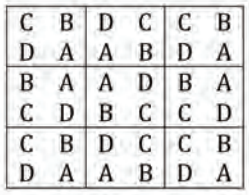
Vol. 54 #4
June 2016

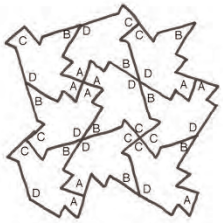
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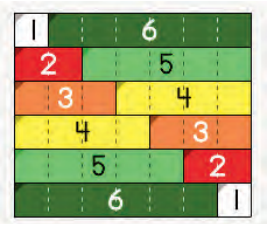
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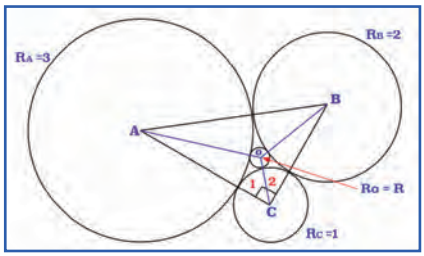




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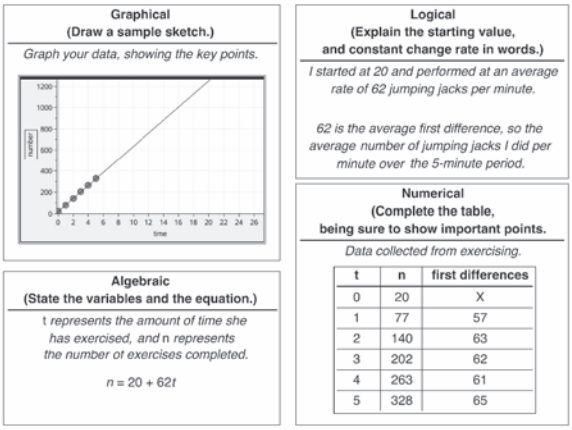


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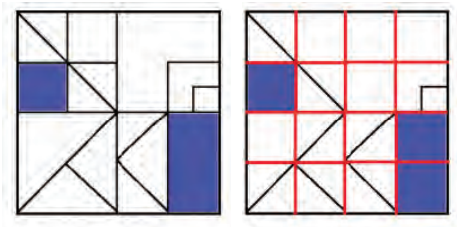


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See In the Middle: Linear Leaping



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Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Sixth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Dan Jarvis, along with **separate files for each of the text, graphics, and/or photographs**. Please email all files to Dan Jarvis at **dan.jarvis@oame.on.ca**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and email address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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The *Ontario Mathematics Gazette*—ISSN 0030-3011—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 40051074.

Editor
▲ **Dan Jarvis**
Email **dan.jarvis@oame.on.ca**

Associate Editors
▲ **Anne Yeager**
Email **anne.yeager@oame.on.ca**
▲ **Marilyn Hurrell**
Email **mhurrell@tbaytel.net**

***Abacus* Co-Editors**
▲ **Mary Lou Kestell**
Email **marylou.kestell@oame.on.ca**
▲ **Kathy Kubota-Zarivnij**
Email **kkz@oame.on.ca**

Copy Editing
▲ **Gitta Berg**
Email **gitta.berg@sympatico.ca**

Design and Production
▲ **Penny Clemens,** Graphic Designer
Email **pennyydezign@gmail.com**

Printing & Binding
▲ **Pole Printing,** Box 69, 1-89 King Street East
Forest, ON N0N 1J0 (519) 786-5112

Advertising Manager
▲ **Robert Sherk**
4366 Snider Rd., Verona, ON K0H 2W0
Home (613) 374-1515
Email **robert.sherk@oame.on.ca**

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Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

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▲ Editor's Message



DANIEL H. JARVIS, O.C.T., Ph.D.
EMAIL: dan.jarvis@oame.on.ca
http://faculty.nipissingu.ca/danj/

Dr. Daniel Jarvis is Professor of Graduate and Mathematics Education in the Schulich School of Education at Nipissing University, North Bay, Ontario. His research interests include instructional technology, integrated curricula, and mathematics of the workplace.

It was unforgivably early on an overcast Wednesday morning that I found myself sitting next to a pleasant stranger on an Air Canada flight bound for sunnier southern climes. My new acquaintance and I eventually—as those confined to contiguous seating in small aircrafts are often wont to do—entered into polite conversation regarding travel destinations, weather, family, and employment. It was during the exchange on this final topic that the issue of Ontario's recently announced *Renewed Math Strategy 2016* (RMS) was brought up by my gregarious co-passenger. It was her opinion that while she certainly approved of the provincial government's investment in her children's education, she was also hopeful that this large sum of money ($60M) would be used wisely, and not just spent on encouraging even more "discovery methods" that had replaced, in her view, the more important "basics"-focused elements of the curriculum. Does this sound familiar to anyone? What are we to make of Canadian media coverage that pits "reformers" versus "traditionalists," revisiting the "math wars" (Wilson, 2003) controversy that lingers still in many parts of North America?

First, let's have a brief look at what the Renewed Math Strategy involves. In her April 4, 2016 news release, Education Minister Liz Sandals announced that "Ontario is dedicating more than $60 million to help support students across the province achieve better results in mathematics." This Strategy, beginning in September 2016, will include: a minimum of 60 minutes per day of protected "learning time for effective mathematics instruction and assessment" in Grades 1–8; up to three math lead teachers in all elementary schools; coaching for principals of select secondary schools to lead improvement; support for learning at home through parent resources; better access to online math resources; math support for Grades 6–8 students outside of the school day; and opportunities for educators to deepen their knowledge in math learning,

teaching, and leading, including a dedicated math Professional Development Day. Furthermore, Minister Sandals claims that the Strategy is "informed by research and the lessons learned from the education sector," and that it focuses on the "needs of students, their families, educators, schools, and district school boards" and "encourages shared responsibility."

After enjoying Canadian educator Marian Small's well-attended NCTM Conference Feature Presenter session entitled *Becoming the Math Teacher They Need!*, I caught up with her for a brief discussion about mathematics education in general, and the Ontario RMS more specifically. Later that evening, I received a requested email from Dr. Small, in which she shared the following related comments, and which I here present to you, with permission:

> I am very happy that the Ministry has decided to remain on the path it has. I believe we are going in the right direction, BUT it takes time and commitment not just from teachers, but from Directors of Education, Superintendents, and particularly principals. We need time and opportunity to help those critical individuals use their roles to move meaningful math, i.e., meaningful for the day and age in which our students live, forward. I believe that each child in Ontario deserves a teacher of math who has a passion and interest in math and who strives to develop a better understanding of how to interpret curriculum expectations to make the learning rich and deep. Having greater expertise in each school by supporting lead teachers is likely to help in this regard. I still believe that the "nut we have to crack" is consolidation of lessons. I remain very committed to the notion that consolidation is not just about sharing strategies. It is about ensuring that the mathematical ideas in which students have engaged are brought to the surface for all students. I also remain convinced that we need to stay the course with open questions.

As I've written in previous publications (Jarvis, 2008, 2016; Jarvis & Franks, 2011), I believe that a "balanced mathematics curriculum" should feature repetition and practice, as well as ample opportunities for hands-on manipulative and technology explorations, and for problem-based learning in both individual and collaborative contexts. Some researchers have insisted that number facts should not be "memorized," practised with flash cards and timed quizzes [Mad Minutes], or demanded through instant recall–common practices which they feel lead to increased student anxiety and disengagement–but rather should be explored through a flexible and rich number sense focus (Boaler & Williams, n.d.; Bratty, 2015). In other jurisdictions, such as the United Kingdom, Alberta, and Manitoba, math curricula explicitly required the memorization of number facts such as the multiplication table (Alphonso, 2014). I suppose I am somewhere in the middle of this particular debate. Knowing one's number facts, in terms of future facility and success in mathematics learning, seems to me to perhaps be more significant than Boaler maintains; yet I do agree that how one comes to understand number sense, and to learn to appreciate the relationships between numbers through modelling and flexible mental strategies, is indeed paramount (e.g., "Making 10" with one's fingers and then with the mind is something I've enjoyed doing with students in Canada, Kenya [see photo below], and Ecuador, and an activity that my teacher education students know I tend to frequently revisit as being foundational. Another one of my favourite activities is the use of non-timed triangle flash cards that allow students to understand the direct relationships between addends/sums and factors/products, as well as see the connections between add/subtract and multiple/divide operations, and the fact family relationships).



At the beginning of Winter term, I had sent an email to my Concurrent B.Ed. classes, asking teacher candidates for their input on what they were observing in Ontario classrooms during practicum placements, in terms of mathematics lessons. I shall just share a few comments here, that relate well to our topic at hand, again with permission. For example, Mary notes the following:

> I was in an FDK placement for three weeks, where math had a specific carpet-time lesson related to math centre activities 2–3 times a week. The teacher used a combination of mini-lessons and multi-modal learning activities to teach probability and ordinal numbers when I was collaborating with her. I am currently in a gifted Grade 4 program. My AT has a strong language and arts background… Though the school has frequent PD on strategies such as the 3-part math lesson, it appears that there isn't a lot of

> support that recognizes the experiences of mature teachers while, still helping them to re-imagine math in the classroom. I know she is frustrated by the types of PD being offered… Her approach has been to use a lot of authentic problem solving.

Another B.Ed. candidate, Sara, describes her observations:

> I attended a workshop along with my Associate Teacher about how to implement it in classrooms throughout all the elementary grades. . . . So for example, a 50-minute math period may begin with a Minds-On activity, followed by a short lesson introducing a new concept, and then students will participate in a 20-minute rotation through five centres: Guided Math/Problem Solving, Shared or Independent Problem Solving, Math Journal, Math Games, and Math Facts. Math Facts always involves numeracy skills of some kind, with the purpose of practising these skills all year, not just during that particular strand… It's a learning process, both for the students and the teachers.

Clearly, the mathematics experiences for students, teachers, and curriculum leaders vary considerably throughout our province, based even on these few brief practicum reports.

The National Council of Teachers of Mathematics (NCTM), in their publication *Principles to Actions: Ensuring Mathematical Success for All* (2014), based on "research-informed actions" (p. 4), argue that in order to move from "pockets of excellence" to "systemic excellence," the mathematics education community in the United States must address certain shortcomings including:

- too much focus is on learning procedures without any connection to meaning, understanding, or the applications that require these procedures
- too many teachers have limited access to the instructional materials, tools, and technology that they need
- too many teachers of mathematics remain professionally isolated, without the benefits of collaborative structures and coaching, and with inadequate opportunities for professional development (p. 3)

Ontario's Renewed Math Strategy has the potential to address the kind of mathematics being learned in classrooms (i.e., proficiency with understanding), the amount of time spent doing so, adequate resources, and professional development supports necessary for teacher and student success. I too remain convinced that we are on the right course of action here in Ontario, and am hopeful that teachers of mathematics, lead teachers, administrators, and coordinators will feel supported by the Ministry, by their respective District School Boards, and by parents/guardians, as the the RMS begins to see implementation this coming Fall.

The June 2016 final Volume 54 issue (V54N4) of the *Gazette* includes three articles, ten regular columns, the elementary school-focused *Abacus* insert, along with several special features.

In their article entitled *A Case for Humility in the Mathematics Classroom*, co-authors Ilona Vashchyshyn, Heidi Neufeld, and Egan Chernoff discuss the three characteristics of humility (a secure, accepting identity; emotional resilience; and openness to new information), two related classroom vignettes, and a series of reflections on why and how this virtue should characterize effective mathematics teaching and learning contexts.

Nipissing University's B.Ed. mathematics education instructor Tim Sibbald and B.Ed. candidate Miranda Wheatstone have collaborated on an intriguing article, *Advancing Escher Art through Generalization*, that extends the problem-solving component of Escher's famous tessellations and illustrates that, in terms of tiering instruction, "it facilitates a diverse array of opportunities for examining cases that show very different pattern behaviours."

Finally, in the first of a 2-part article, a retired teacher, pilot, and skydiver, André Lemaire, explores different mathematical methods for solving the ancient Greece classic "Three Spheres of Radii 3, 2, and 1 Mutually Tangent Problem," including the use of Pythagoras's Theorem, Heron's Formula, trigonometry, Cartesian coordinates, and analytic geometry.

Regular columns include the following highlights: OAME President, Tim Sibbald (in his final President's Message), discusses the NCTM Board of Directors meeting that was recently held in Toronto, and during which the strengths of, and current challenges faced by, Ontario mathematics education were discussed; Todd Romiens (OAME/NCTM Report) highlights NCTM's new Innov8 Conference designed to specifically help teachers engage struggling learners, and NCTM's STEM Project (science, technology, engineering, mathematics) which continues to inform the American Congress; Lynda Colgan (Hey, It's Elementary) discusses the Renewed Math Strategy, with special emphasis on what she refers to as the "Lead Teacher dilemma"; Ross Isenegger, Agnes Grafton, Markus Wolski, and Greg Clarke (Provincial Digital Learning Resources) continue explaining the various applications of the **mathies.ca** Relational Rods resource (e.g., reasoning and

proving, even/odd numbers, summing the natural numbers, partitions, and the ability to annotate within the software); and Mary Bourassa (Technology Corner) explores Nat Banting's website "Fraction Talks," and shares several examples of how others have used this helpful online resource in their mathematics teaching.

Carly Ziniuk (*In the Middle*) presents an investigation entitled "Linear Leaping," in which linear relationships connected to healthy movement are examined by students using available data and a 4-part template which includes graphical, logical, numerical, and algebraic thinking; Assessment Abby (eponymous) focuses on linking assessment and instruction; Shawn Godin (What's the Problem?) analyzes two rich problems involving card picking, probability, and the classic Monty Hall Problem with opened/closed doors and hidden prizes; Ann Kajander (MB4T) examines her fourth and final case involving division of integers, that of a positive divided by a negative number; and in the *Abacus* insert, co-editors Mary Lou Kestell and Kathy Kubota-Zarivnij complete their Volume 54 focus on relational thinking, here focusing on the operation of division.

Volume 54 Issue 4 also includes several special features: (i) a David Davidson Tribute (1940–2016), written by his close friends Dean Murray and Jack Weiner; (ii) a Letter to the Editor by Tim Sibbald, which raises awareness about a forthcoming publication and opportunities for participation; (iii) three further Ontario mathematics education researcher profile highlights; and (iv) some final excerpts from Lerman's *Encyclopedia of Mathematics Education* (2014) regarding a variety of interesting, math-related issues.

It was my great pleasure to attend the NCTM "Building Bridges to Student Success" Conference 2016 in San Francisco, California, and to accept, on behalf of OAME, the Publication Award from NCTM President, Diane Briars, during the annual meeting of affiliate delegates.



Once again, I would just like to take a final moment to thank OAME for allowing me this great privilege to serve as your Editor for the past two volumes. In my view, Presidents Paul Alves and Tim Sibbald have both been wonderful advocates for mathematics education, providing the organization with informed and indefatigable leadership. My thanks again to my gracious predecessor, Immaculate Namukasa, and I look forward to paying her kindness forward as I begin to work with my successor, Amy Lin, who will take over as *Gazette* Editor this summer.

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Errata: *In the approximately 200 printed copies of V54N3 (i.e., we were able to correct it in the digital version) paragraph three of the Editorial read as "(10 raised to the power of a googol, or a 1 followed by a 1000 zeroes)," but should have been "(10 raised to the power of a googol, or a 1 followed by a googol of zeroes)". Similarly, "(10 raised to the power of a googolplex, or a 1 followed by 10 000 zeroes)," should have read "(10 raised to the power of a googolplex, or a 1 followed by a googolplex of zeroes)".*